Umkehrfunktionen der quadratischen Funktionen

Die quadratischen Funktionen sind nicht auf der ganzen Definitionsmenge $D = \mathbb{R}$ umkehrbar. Die Abszisse des Scheitelpunktes bestimmt die Aufteilung der Definitionsmenge.

Aufgabe 1

$y = f(x) = 0.5x^2$ $\qquad D_f = \mathbb{R}$, Scheitelpunkt S(0/0), $W_f = \mathbb{R}_0^+$

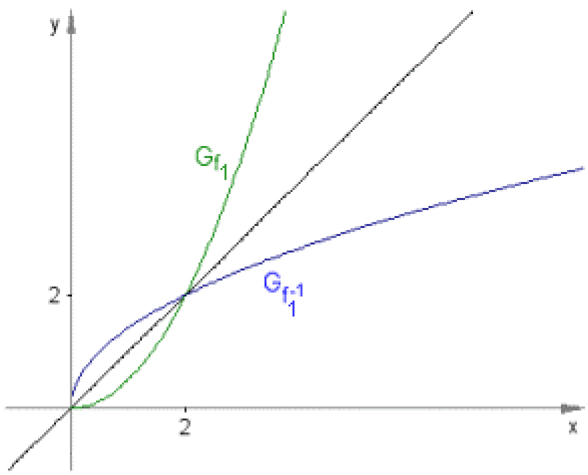
$D_{f1} = \mathbb{R}_0^+$, $W_{f1} = W_f$ $\qquad\qquad D_{f2} = \mathbb{R}_0^-$, $W_{f2} = W_f$

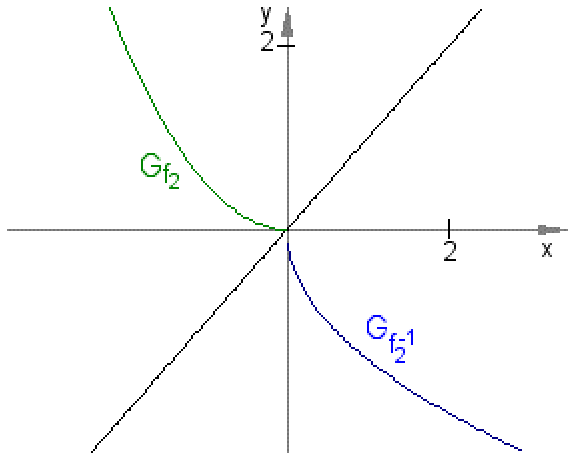
für f^{-1}:

$$x = 0.5y^2$$
$$y^2 = 2x$$
$$|y| = \sqrt{2x}$$

$y = f_1^{-1}(x) = \sqrt{2x}$ $\qquad\qquad y = f_2^{-1}(x) = -\sqrt{2x}$

$D_{f1^{-1}} = W_{f1}$, $\quad W_{f1^{-1}} = D_{f1}$ $\qquad\qquad D_{f2^{-1}} = W_{f2}$, $\quad W_{f2^{-1}} = D_{f2}$





Aufgabe 2

$y = f(x) = 4 - x^2$ $\qquad D_f = \mathbb{R}$, Scheitelpunkt S(0/4), $W_f =]\leftarrow, 4]$

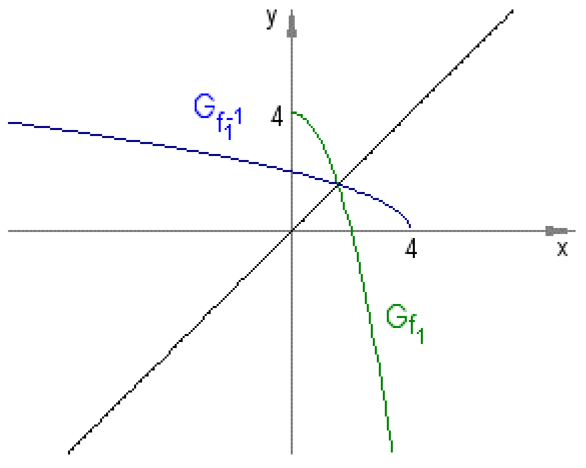
$D_{f1} = \mathbb{R}_0^+$, $W_{f1} = W_f$ $\qquad\qquad D_{f2} = \mathbb{R}_0^-$, $W_{f2} = W_f$

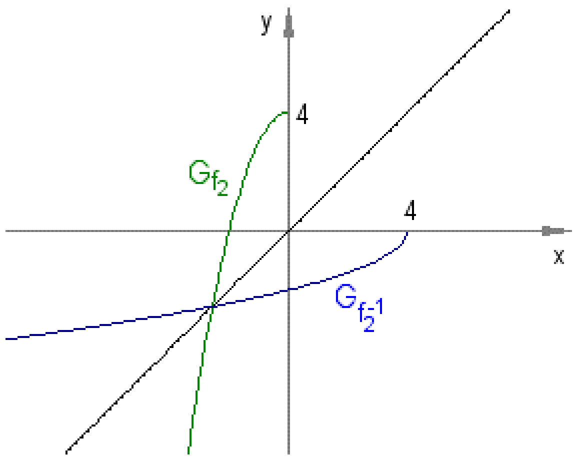
für f^{-1}:

$$x = 4 - y^2$$
$$y^2 = 4 - x$$
$$|y| = \sqrt{4 - x}$$

$y = f_1^{-1}(x) = \sqrt{4 - x}$ $\qquad\qquad y = f_2^{-1}(x) = -\sqrt{4 - x}$

$D_{f1^{-1}} = W_{f1}$, $\quad W_{f1^{-1}} = D_{f1}$ $\qquad\qquad D_{f2^{-1}} = W_{f2}$, $\quad W_{f2^{-1}} = D_{f2}$





Aufgabe 3

$y = f(x) = (x - 3)^2$ $D_f = \mathbb{R}$, Scheitelpunkt S(3/0), $W_f = \mathbb{R}_0^+$

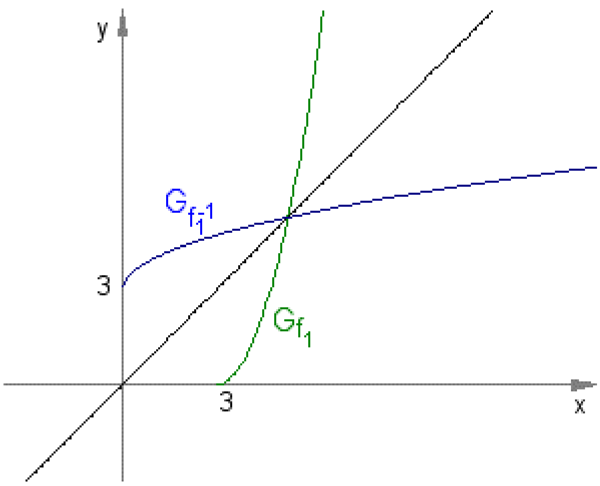
$D_{f1} = [3, \rightarrow [$, $W_{f1} = W_f$ $D_{f2} =]\leftarrow, 3]$, $W_{f2} = W_f$

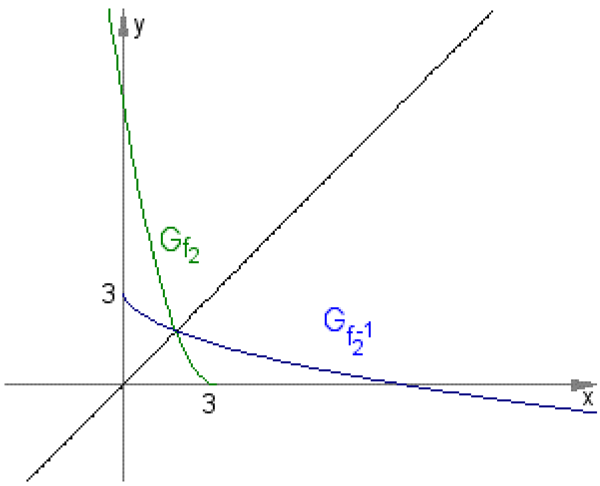
für f^{-1}:

$$x = (y - 3)^2$$
$$|y - 3| = \sqrt{x}$$

$y = f_1^{-1}(x) = 3 + \sqrt{x}$ $y = f_2^{-1}(x) = 3 - \sqrt{x}$

$D_{f1^{-1}} = W_{f1}$, $W_{f1^{-1}} = D_{f1}$ $D_{f2^{-1}} = W_{f2}$, $W_{f2^{-1}} = D_{f2}$





Aufgabe 4

$y = f(x) = 0.5x^2 + x - 0.5$ $\qquad D_f = \mathbb{R}$
für Scheitelpunkt: $y = 0.5(x^2 + 2x) - 0.5 = 0.5((x + 1)^2 - 1) - 0.5 = 0.5(x + 1)^2 - 1$
$S(-1/-1)$, $W_f = [-1, \rightarrow [$

$D_{f1} = [-1, \rightarrow [$, $W_{f1} = W_f$ $\qquad D_{f2} =] \leftarrow , -1]$, $W_{f2} = W_f$

für f^{-1}:

$$x = 0.5(y + 1)^2 - 1$$
$$(y + 1)^2 = 2(x + 1)$$
$$|y + 1| = \sqrt{2(x+1)}$$

$y = f_1^{-1}(x) = -1 + \sqrt{2(x+1)}$ $\qquad y = f_2^{-1}(x) = -1 - \sqrt{2(x+1)}$

$D_{f1^{-1}} = W_{f1}$, $W_{f1^{-1}} = D_{f1}$ $\qquad D_{f2^{-1}} = W_{f2}$, $W_{f2^{-1}} = D_{f2}$